Entrenchment and Retractability: A Preliminary Report

Abhaya Nayak, Maurice Pagnucco, Norman Foo and Rex Kwok

Knowledge Systems Group
Basser Deptartment of Computer Science (F09)
University of Sydney
NSW 2006, Australia
email: {abhaya|morri|norman|rkwok}@cs.su.oz.au

Abstract

In the coherentist approach to belief change, all beliefs are assumed to be ordered according to their epistemic entrenchment. On the other hand, in the foundational approach to belief change, which explicitly distinguishes "basic" beliefs from "inferred" beliefs, basic beliefs are ordered according to their retractability. Both entrenchment and retractability are distinct tie-breaking mechanisms, although there is a suggestion in the literature that they might determine each other to some degree. Some writers, such as Neil Tennant, confuse these two mechanisms and, as a result, thoroughly confound the issue of belief change. In this paper we offer some results of our investigation into the interplay between entrenchment and retractability.

Keywords: Belief (Base) Change, Contraction, Entrenchment, Retractability.

1 Introduction

The approaches to belief change may be broadly classified into two classes: those that examine the dynamics of belief sets (the set of sentences that an agent is committed to believe) and the dynamics of belief bases (the set of sentences that represent the agent's "basic" beliefs). The former is often referred to as the coherentist approach, whereas the latter is referred to as the foundational approach.

One of the arguments traditionally offered in support of the belief base approach is the following. The difference between basic beliefs and inferred beliefs is important. Hence, in the representation of a belief set, this distinction should be explicitly maintained, as is done in the belief base approach. However, in the coherentist approach this important distinction is obliterated. In a very interesting paper [5], Peter Gärdenfors contested this argument: he argued that the coherentist approach to belief dynamics takes into account two factors when determining the new belief set after receiving an input, namely, the current belief set and a tie-breaking mechanism such as epistemic entrenchment. He argued that although a belief set itself does not make a distinction between basic beliefs and inferred beliefs, the accompanying entrenchment relation often contains such information. But it has never been settled as to whether an entrenchment relation accompanying a belief set can uniquely determine a base for that belief set. Conversely, we do not know to what extent a belief base determines the associated entrenchment either.

As epistemic entrenchment is used as a tie-breaking mechanism in the coherentist approach, the relation of retractability is used as a tie-breaking mechanism in the foundational approach. Although these are distinct relations, there is a suggestion in the literature (see, e.g., [7]) that they determine each other to some extent. The connection between these two mechanisms has not been rigorously studied in the literature. It is our view that the proper investigation of

Gärdenfors's proposal can be carried out only by examining how the entrenchment of beliefs and retractability of the corresponding basic beliefs influence each other.

This investigation is important for one more reason. A recent paper by Neil Tennant [13], which launches a severe attack on the current state of belief change research, is replete with confusion between entrenchment and retractability (see Hansson and Rott's reply [8] to Tennant). This is symptomatic of a certain deficiency in the literature. It is hoped that our investigation will result in the eradication of that deficiency to some extent.

In section §2 we will give a quick review of the necessary background. Section §3 contains some of our preliminary findings. We conclude with a brief discussion in section §4.

2 Background

Our ultimate goal is to investigate to what extent the choice of a belief base is determined by the epistemic entrenchment of beliefs in a coherentist approach, and to what extent the entrenchment of beliefs is determined by the choice of a belief base. To carry out this investigation profitably, we must fix at least two parameters: (1) what is the nature of coherentist belief change, and (2) what is the nature of a foundational belief change. It is not necessary to go to all three kinds of belief change, namely, expansion, contraction and revision [3]; focussing on belief contraction is sufficient for our purpose.

There are many brands of coherentist and foundational belief contraction. For instance, in the (coherentist) AGM approach [1, 3] recovery is satisfied by the belief contraction operation, but it is not satisfied by a coherentist model of belief contraction propounded by Isaac Levi [9]. Similarly, Nebel [11], Nayak [10], Hansson [6, 7] and Fuhrmann [2] have examined different approaches to foundational belief contraction. A fruitful exploration of our problem can be performed only if the coherentist and foundational belief contraction operations selected for our purpose behave in a similar fashion. In short of such parity, it is not clear what lessons can be learnt from a comparison.

We will take our coherentist belief contraction to be AGM contraction. For the foundational contraction, we will consider the account of belief contraction examined by Nayak [10]. Our choice of the latter is based on the fact that, like the AGM contraction operation, it satisfies all the Gärdenfors (rationality) postulates of belief contraction (see [3] for a complete list of these postulates). In this section we briefly outline these two systems.

2.1 AGM Belief Contraction

In the AGM approach, the object language (the language in which the beliefs of an agent are represented) is a propositional language $\mathcal{L}$ closed under the usual connectives $\neg$, $\rightarrow$, $\leftrightarrow$, $\wedge$ and $\vee$. Two eminent members of $\mathcal{L}$ are $\top$ (Truth) and $\bot$ (Falsity). The background logic (the logic of the agent) is represented by its consequence operation Cn which satisfies the following conditions: For all sets of sentences Γ, Γ' and all sentences x and y,

Inclusion:	$\Gamma \subseteq Cn(\Gamma)$
Iteration:	$Cn(Cn(\Gamma)) = Cn(\Gamma)$
Monotonicity:	$Cn(\Gamma) \subseteq Cn(\Gamma')$ whenever $\Gamma \subseteq \Gamma'$
Supraclassicality:	$x \in Cn(\Gamma)$ if Γ classically implies x
Deduction:	$y \in Cn(\Gamma \cup \{x\})$ iff $(x \rightarrow y) \in Cn(\Gamma)$
Compactness:	If $x \in Cn(\Gamma)$ then $x \in Cn(\Gamma')$ for some finite $\Gamma' \subseteq \Gamma$.

We often write $\Gamma \vdash x$ for $x \in Cn(\Gamma)$.

Any set of sentences $K \subseteq \mathcal{L}$ closed under Cn is called a belief set (or theory) and represents a possible belief state. We may interpret the members of K to be the sentences that the agent holds as beliefs. We use $K_\perp$ to denote the absurd belief set $\mathcal{L}$.

In order to construct the belief contraction operation, a belief set K is required to be accompanied by a tie-breaking mechanism. An epistemic entrenchment relation $\preceq$ is such a mechanism. This relation is defined over the members of $\mathcal{L}$, and is relative to the belief set K. The following are the *standard* conditions that the relation $\preceq$ is required to satisfy: where $x,\ y$ and z are arbitrary sentences of $\mathcal{L}$,

$(SEE1)$	If $x \preceq y$ and $y \preceq z$ then $x \preceq z$	(transitivity)
$(SEE2)$	If $x \vdash y$ then $x \preceq y$	(dominance)
$(SEE3)$	For any x and y, $x \preceq x \wedge y$ or $y \preceq x \wedge y$	(conjunctiveness)
$(SEE4)$	When $K \neq K_\perp,\ x \notin K$ iff $x \preceq y$ for all y	(minimality)
$(SEE5)$	If $y \preceq x$ for all y, then $\vdash x$	(maximality)

The inequality $x \preceq y$ may be interpreted as saying that it is not more difficult to give up x than it is to give up y. The motivation behind the constraints (SEE1)-(SEE5) can be found in Gärdenfors[3] (§4.6).

A theory contraction operation $-_\preceq$ is constructed from $\preceq$ in the following manner:

(SEE$-$): $\qquad y \in K_x^{-\preceq}$ iff both $y \in K$ and either $x \prec (x \vee y)$ or $\vdash x$

where $\prec$ is the strict part of the relation $\preceq$. Gärdenfors and Makinson[4] have shown that if $\preceq$ satisfies (SEE1)–(SEE5), then the corresponding contraction operation satisfies a set of rationality postulates called the Gärdenfors postulates of belief contraction, and in turn, that every relation satisfying (SEE1)-(SEE5) can be defined from some contraction operation that satisfies the Gärdenfors postulates.

2.2 Foundational Belief Contraction

In [10] Nayak offers an account of foundational belief contraction that is as well behaved as the AGM account of belief contraction. Whereas in the coherentist approach a closed belief set represents the beliefs of an agent, in this account an open set of sentences represents the basic (explicit) beliefs of the agent. Instead of the entrenchment relation, in this account a choice function C over 2^B, called a rejector, accompanies B. This function chooses the worst members of any subset of B. It is assumed to satisfy the following conditions: For all subsets X, Y of $B,$

(1)	If $X \neq \emptyset$, then $C(X) \neq \emptyset$;	(Success)
(2)	If $x \in C(X)$ and $y \in C(X)$, then $x = y$;	(Credulity)
(3)	$C(X) \cap Y \subseteq C(X \cap Y)$;	(α, Chernoff)

We will assume that the base B for a belief set K is finite. In order to facilitate the comparison between (entrenchment based) coherentist belief contraction and foundational belief contraction, instead of the choice function C, we will use the following relation, $<$, to be called the relation of retractability (the name is due to Fuhrmann [2]) defined in terms of C:

Definition 1 $x < y$ *iff* $x \in C(\{x, y\})$.

The sentence $x < y$ may be interpreted as saying that x is easier to retract than y. Theorem 1 characterizes the relation of retractability thus defined.

Theorem 1 *When C satisfies Success, Credulity and Chernoff, the relation $<$ defined above is transitive, antisymmetric and connected, i.e., for all sentences x, y and z in B, (1) If $x < y$ and $y < z$, then $x < z$, (2) If $x < y$ and $y < x$, then $x = y$, and (3) Either $x < y$ or $y < x$.*

Hence given a belief base B, every subset of it has unique $<$-minimal and $<$-maximal elements.[1] We will refer to the $<$-minimal element of B as b_1. By b_{i+1} we will denote the $<$-minimal element of $B \setminus \{b_1, \ldots, b_i\}$. By b_n we will refer to the last element in the sequence $b_1, b_2 \ldots$, namely, the $<$-maximal element of B.

Given this definition, the base contraction operation $\ominus_<$ (see definitions 5.3, 5.8 and discussion thereafter in [10]; the operation $\ominus_<$ here stands for the operation $\ominus_m$ in that work) can be constructed in the following manner (note the use of backward induction in this definition):

Definition 2 *If $\vdash a$, then $(B \ominus_< a) = B$. Otherwise,*

$$b_i \in B \ominus_< a \text{ iff } \begin{cases} \{b_n\} \nvdash a & \text{if } i{=}n \\ (\{b_{i+1}, b_{i+2}, \ldots, b_n\} \cap B \ominus_< a) \cup \{b_i\} \nvdash a & \text{otherwise} \end{cases}$$

Notation: *From here onwards, we will assume that the members of B are ordered by $<$ in ascending order ($\langle b_1, \ldots, b_n \rangle$) and refer to it as $\langle b_i \rangle$. We will call $\langle b_i \rangle$ a prioritized belief base; more precisely, the (retractability) prioritization of B. Instead of the contracted set $B \ominus_< a$, we will henceforth write $\langle b_i \rangle_a^\ominus$. Note that the subscript $<$ is omitted here, since the ordering $<$ is implicit in the notation. This abbreviation will apply to other operations, namely, $\dot{-}_<$ and $-_<$ to be introduced in definitions 3 and 4, as well.*

Example:[2]
$$\langle a \leftrightarrow b, a \vee b \rangle_a^\ominus = \{a \vee b\}$$
$$\langle a \vee b, a \leftrightarrow b \rangle_a^\ominus = \{a \leftrightarrow b\}$$

The contraction operation $\ominus$ often results in excessive loss of information. For instance, $\langle a \wedge b \wedge c \rangle_a^\ominus = \emptyset$. In order to amend this situation, a somewhat weaker base contraction operation, $\dot{-}_<$ is introduced (see definition 5.4 in [10]) as follows:

Definition 3 $\langle b_i \rangle_a^{\dot{-}} = \langle b_i \rangle_a^\ominus \cup \{a \rightarrow b_j | 1 \leq j \leq n,\ b_j \notin \langle b_i \rangle_a^\ominus\}$

Example:
$$\langle a \leftrightarrow b, a \vee b \rangle_a^{\dot{-}} = \{a \vee b, a \rightarrow (a \leftrightarrow b)\} = \{a \vee b, a \rightarrow b\}^3$$
$$\langle a \vee b, a \leftrightarrow b \rangle_a^{\dot{-}} = \{a \leftrightarrow b, a \rightarrow (a \vee b)\} = \{a \leftrightarrow b, \top\}.$$

From the operation $\dot{-}_<$, a foundational belief (set) contraction operation $-_<$, for contracting a belief set $K_{\langle b_i \rangle}$ with base $\langle b_i \rangle$, may be defined as follows:

Definition 4 $(K_{\langle b_i \rangle})_a^- = Cn(\langle b_i \rangle_a^{\dot{-}})$

Example:
$$(K_{\langle a \leftrightarrow b, a \vee b \rangle})_a^- = Cn(\{b\})$$
$$(K_{\langle a \vee b, a \leftrightarrow b \rangle})_a^- = Cn(\{a \leftrightarrow b\})$$

It is shown in [10] that this foundational belief contraction satisfies the Gärdenfors postulates of contraction. We use this operation as a benchmark in our investigation.

---

[1] x is a $<$-minimal element of a set X iff $x < x'$ for all $x' \in X$. On the other hand, x is a $<$-maximal element of a set X iff $x' < x$ for all $x' \in X$.

[2] For simplicity, the examples in this section assume that a and b are logically independent of each other.

[3] This operation is insensitive to syntactic difference; hence $a \rightarrow (a \leftrightarrow b)$ can be replaced by $a \rightarrow b$ without any unintended side-effect.

3 Some Technical Results

In the coherentist setting, a belief state is represented by a set of beliefs ordered by the relation of epistemic entrenchment. In the foundational setting, on the other hand, a belief state is represented as a set of *basic* beliefs ordered by their retractability. We assume that two belief states are dynamically equivalent if and only if they result in the same set of beliefs after contraction by any arbitrary sentence. In the coherentist framework we will consider $-_{\preceq}$ to be the contraction; in the foundational framework, we will use $-_{<}$.

Notation: *We will adopt the following notation from Rott ([12]) to represent a given entrenchment relation. Let $\preceq$ be an entrenchment relation that partitions $\mathcal{L}$ into a finite number, $n+2$, of equivalence classes. (By SEE4 and SEE5, there are at least two equivalence classes, the $\preceq$-minimal and $\preceq$-maximal classes, which are different.) Denote by s_{n+1} the sentence $\top$. Denote by s_i, $0 \le i \le n$ the sentence $s_{i+1} \wedge \bigwedge(\{x \mid x \prec s_{i+1}$*and, for no y does it hold that $x \prec y \prec s_{i+1}\}$. *It is easily noticed that $s_0 = \bot$. We will succinctly represent the relation $\preceq$ as $\bot = s_0 \prec s_1 \prec \ldots \prec s_n \prec s_{n+1} = \top$.*

First we offer a general argument to the effect that an entrenchment relation indeed encodes some information about the basic beliefs. Consider the entrenchment relation $\bot \prec s_1 \prec \ldots \prec s_n \prec \top$ associated with the belief set K. Now, assuming the consistency of the associated belief set, it follows from SEE4 that a sentence x is believed iff $\bot \prec x$. Note that $s_1 \vdash s_i$ for all $1 \le i$. Furthermore, $\{s_1, \ldots, s_n\} \vdash s_1$. Hence $\{s_1\}$ can function as a belief base for the belief set in question. (The retractability relation associated with this base is unique and trivial.) So can the base $\{p \vee s_1, \neg p \vee s_1\}$, where p is an arbitrary sentence independent of s_1. Now, either $1 < n$ or $n = 1$, and in either case some base or other for K is disallowed by the given entrenchment relation. So long as $1 < n$, $\{s_1\}$ is not an appropriate belief base. (For instance, as easily shown, $s_2 \in K_{s_1}^{-\preceq}$, but $s_2 \notin (K_{\langle s_1 \rangle})_{s_1}^{-}$.) On the other hand, if $n = 1$, neither of the prioritized bases $\langle p \vee s_1, \neg p \vee s_1 \rangle$ and $\langle \neg p \vee s_1, p \vee s_1 \rangle$ will do. (For instance, in this case, $K_{s_1}^{-\preceq} = Cn(\emptyset)$ whereas $(K_{\langle p \vee s_1, \neg p \vee s_1 \rangle})_{s_1}^{-} = Cn(\{\neg p \vee s_1\})$.) Thus, there are always (prioritized) belief bases excluded from consideration given an entrenchment relation. To that extent, an entrenchment relation encodes some information about the belief base.

One might wonder whether there always are foundational belief states that show the same dynamic behavior as a given coherent belief state. Theorem 2 below shows that this is indeed so: given any entrenchment relation $\preceq$ with belief set K, we can construct a prioritized belief base $\langle b_i \rangle$ for K such that $K_x^{-\preceq} = (K_{\langle b_i \rangle})_x^{-}$ for every sentence x.

Definition 5 *A* canonical belief base *for an entrenchment relation $\preceq = \bot \prec s_1 \prec \ldots \prec s_n \prec \top$ is defined to be the set $B_{\preceq} = \{s_1, s_2, \ldots s_n\}$.*

Note that the members of this belief base, not being ordered by any retractability relation, are subject to different orderings. An interesting way to order them is to let the retractability relation mimic the entrenchment in question. We call the result of ordering $B_{\preceq}$ in that manner a *canonically prioritized belief base* for the entrenchment relation $\preceq$.

Definition 6 *A canonically prioritized belief base for the entrenchment relation $\preceq = \bot \prec s_1 \prec \ldots \prec s_n \prec \top$ is defined to be the sequence $\langle s_i \rangle$.*

Theorem 2 *Let K be any belief set, and $\preceq$ an entrenchment relation for it. Let $\langle s_i \rangle$ be the canonically prioritized belief base for $\preceq$. Then, $K_x^{-\preceq} = (K_{\langle s_i \rangle})_x^{-}$ for all sentences x.*

This result makes explicit an interesting connection between entrenchment and retractability. However one should not be carried away with this apparent similarity, for, the canonically prioritized belief base is not the only prioritized base that will show this type of behavior. Theorem 3 below shows that a canonical belief base is priority-insensitive with respect to $\ominus_<$-contraction. (The analogous result for foundational contraction $-_<$ would easily follow as a corollary.) Hence, different foundational belief states (namely, different prioritizations of $B_\preceq$) are dynamically equivalent to a given coherent belief state (K ordered by $\preceq$).

Theorem 3 *Let K be any belief set, and $\preceq$ an entrenchment relation for it. Let $\langle b_i \rangle$ and $\langle b'_i \rangle$ be two different (retractability) prioritizations of its canonical belief base $B_\preceq$. Then, $(K_{\langle b_i \rangle})^\ominus_x = (K_{\langle b'_i \rangle})^\ominus_x$ for every sentence x.*

Note that given an entrenchment relation $\preceq$, the canonical belief base $B_\preceq$ admits too many redundant members. In particular, when $B_\preceq = \{s_1, s_2, \ldots s_n\}$ is constructed in the manner prescribed earlier (see definition 5), the members $s_2, \ldots, s_n$ are redundant, since each of them is entailed by s_1. One may suspect, in light of Theorem 3, that the reason why the retractability relation $<$ does not play any role here is that in presence of the strong logical dependence among the members of B, the extralogical relation, $<$ becomes insignificant. This would suggest that when a base is purged of redundance completely, the role of $<$ will be more pronounced. This is indeed so, and is demonstrated by Theorem 4.

Theorem 4 *Let B be any belief base each of whose members is logically independent of the rest, i.e., for all different $b \in B,\ B \setminus \{b\} \nvdash b$. Let $\langle b_i \rangle$ and $\langle b'_i \rangle$ be any two different retractability prioritizations of B. Then there is a sentence x such that $\langle b_i \rangle^\ominus_x \neq \langle b'_i \rangle^\ominus_x$.*

Example: *Let $a,\ b$ be two sentences independent of each other. Consider the two prioritized bases $\langle a, b \rangle$ and $\langle b, a \rangle$. Now, $\langle a, b \rangle^\ominus_{a \wedge b} = \{b\}$ and $\langle b, a \rangle^\ominus_{a \wedge b} = \{a\}$, from which the desired result easily follows.*

We note the asymmetry here: although a canonical belief base $B_\preceq$ is insensitive to retractability (with respect to contraction), the belief set K is not insensitive to $\preceq$. The same belief set K under different entrenchment relations will, in general, exhibit different contraction behaviour.

Theorems 3 and 4 are about extremes of the spectrum. In general, a base is not as full of redundancy as presupposed by Theorem 3; neither are its members independent of the rest. Hence the relation of retractability is, in general, not completely pointless, as Theorem 3 would lead one to suppose; on the other hand, its role is over-emphasized in Theorem 4. Our final result, Theorem 5, applies to a more general situation.

Definition 7 *Let $\langle b_i \rangle$ and $\langle b'_i \rangle$ be two prioritized belief bases of the same cardinality n. We will call $\langle b_i \rangle$ and $\langle b'_i \rangle$ "twins" just in case (1) For no $i \neq i'$ is it true that $\vdash b_i \leftrightarrow b_{i'}$, and (2) there are numbers $1 = p_1 < p_2 < \ldots < p_m = n+1$, such that for each $j < m$ it holds both that (2a) $b_{p_j}, b_{p_j+1}, \ldots, b_{p_{j+1}-1}$ is a permutation of $b'_{p_j}, b'_{p_j+1}, \ldots, b'_{p_{j+1}-1}$, and (2b) both $\{b_{p_j}, b_{p_j+1}, \ldots, b_{p_{j+1}-1}\}$ and $\{b'_{p_j}, b'_{p_j+1}, \ldots, b'_{p_{j+1}-1}\}$ are connected under $\vdash$.*[4]

This definition essentially says that two prioritized belief bases $\langle b_i \rangle$ and $\langle b'_i \rangle$ are twins if and only if they can be (parallely) partitioned at points $p_j,\ j < m$ such that the corresponding

[4]A set X is connected under $\vdash$ just in case either $x \vdash x'$ or $x' \vdash x$ for all $x, x' \in X$.

parts are permutable and are connected under $\vdash$. Note here that if two prioritized belief bases are identical then, trivially (given this definition), they are twins. In order to appreciate the connection of this definition with earlier results, note that every canonical belief base is a set of sentences whose members do not repeat (modulo logical equivalence) and are connected under $\vdash$. Furthermore, every set with such properties is a canonical belief base for some entrenchment relation.

Example: *Let $\langle b_i \rangle$ be $\langle d, e \wedge f, e \vee f, e, g, h \rangle$ and $\langle b'_i \rangle$ be $\langle d, e, e \vee f, e \wedge f, g, h \rangle$. These two prioritized belief bases are twins. This can be verified by setting $p_1 = 1, p_2 = 2, p_3 = 5, p_4 = 6, p_5 = 7$. The partitions of $\langle b_i \rangle$ and $\langle b'_i \rangle$, respectively, are $\{\{d\}, \{e \wedge f, e \vee f, e\}, \{g\}, \{h\}\}$ and $\{\{d\}, \{e, e \vee f, e \wedge f\}, \{g\}, \{h\}\}$. Note here that a singleton-sequence is, trivially, a permutation of itself, and is connected under $\vdash$. Since $\{e \wedge f, e \vee f, e\}$ is a permutation of $\{e, e \vee f, e \wedge f\}$, and is connected under $\vdash$, the desired result follows.*

Theorem 5 *If two prioritized belief bases $\langle b_i \rangle$ and $\langle b'_i \rangle$ are twins, then $\langle b_i \rangle^{\ominus}_x = \langle b'_i \rangle^{\ominus}_x$ for every sentence x.*

This theorem offers a general condition under which a belief base under different retractability relations exhibit identical contraction behavior. We continue the preceeding example in order to show an application of this theorem.

Example: *Let $\langle b_i \rangle$ and $\langle b'_i \rangle$ be twins as in the earlier example. Assume for simplicity that d, e, f, g and h are atomic sentences. (The reader is invited to dispense with this simplifying assumption and verify the more complicated situation.) In accordance with Theorem 5, it follows that $\langle b_i \rangle^{\ominus}_x = \langle b'_i \rangle^{\ominus}_x$ for every sentence x. We verify this result for two different values of x. It is easily verified that $\langle b_i \rangle^{\ominus}_{f \wedge g} = \{d, e \vee f, e, g, h\} = \{d, e, e \vee f, g, h\} = \langle b'_i \rangle^{\ominus}_{f \wedge g}$. Similarly, $\langle b_i \rangle^{\ominus}_{f \vee g} = \{d, e \vee f, e, h\} = \{d, e, e \vee f, h\} = \langle b'_i \rangle^{\ominus}_{f \vee g}$.*

We leave open the question whether twinship among the (foundational) belief states is a necessary condition for such invariance in contraction behavior. Note also that this result does not presuppose a belief base connected under $\vdash$. Hence, although twins of a given prioritized belief base are very likely to correspond to the same entrenchment relation, we are yet to know how such an entrenchment relation is to be constructed. Once a recipe for this construction is found, we would achieve substantial progress in the direction of resolving our original problem.

4 Conclusion

We started with the goal of discovering how the two tie-breaking mechanisms used in the coherentist and foundational approaches to belief change influence each other. We decided to confine the discussion to the contraction operation since the results apply, *mutatis mutandis,* to revision. In §2 we briefly described the coherentist and foundational contraction operations of our choice. Our choice was dictated by the fact that the two operations chosen should be equally well behaved. In section §3 we presented some of our findings. In particular, we showed that (1) there is a method for constructing a foundational belief state from a given coherentist belief state such that the two states are dynamically equivalent, (2) if a belief base consists of sentences that exhibit excessive logical dependence, then it is insensitive to prioritization, (3) if a belief base consists of sentences that are logically independent of the rest, then the base is maximally affected by their prioritization, and (4) belief bases that can be partitioned at some points in such a manner that the corresponding parts are permutations of each other, and are connected under $\vdash$, show identical contraction behavior.

It would be desirable to find the exact conditions under which a coherent and a foundational belief state are dynamically equivalent. While we have made some progress in this direction, a complete characterization of such equivalence is yet to be obtained. This is being pursued.

References

[1] Alchourrón, C. E., P. Gärdenfors, and D. Makinson, "On the Logic of Theory Change: Partial Meet Contraction and Revision Functions" *Journal of Symbolic Logic,* vol. 50, pp.510-530, 1985.

[2] Fuhrmann, André, "Theory Contraction through Base Contraction," *Journal of Philosophical Logic,* vol. 20, pp. 175-203, 1991.

[3] Gärdenfors, Peter, *Knowledge in Flux. Modeling the Dynamics of Epistemic States,* Bradford/MIT, Cambridge (Massachussettes), 1988.

[4] Gärdenfors, Peter and David Makinson, "Revision of Knowledge Systems using Epistemic Entrenchment," in *Proceedings of the Second Conference on Theoretical Aspects of Reasoning about Knowledge,* M. Vardi ed., Los Altos, CA: Morgan Kaufmann. pp. 83-95, 1988.

[5] Gärdenfors, Peter, "The Dynamics of Belief Systems: Foundations vs. Coherence Theories," *Revue Internationale de Philosophie,* Vol. 44, pp. 24-46, 1990.

[6] Hansson, Sven Ove, "New Operators for Theory Change," *Theoria,* vol. 55, pp. 114-136, 1989.

[7] Hansson, Sven Ove, "A Dyadic Representation of Belief", in Peter Gärdenfors (ed.) *Belief Revision*, CUP, pp. 89-121, 1992.

[8] Hansson, Sven Ove and Hans Rott, "How Not to Change the Theory of Theory Change", forthcoming, *British Journal of Philosophy of Science*.

[9] Levi, Isaac, *Fixation of Belief and its Undoing,* Cambrdige University Press, 1991.

[10] Nayak, Abhaya C., "Foundational Belief Change", *Journal of Philosophical Logic*, vol. 23, pp. 495-533, 1994.

[11] Nebel, Bernhard, "Belief Revision and Default Reasoning: Syntax Based Approaches," in J. A. Allen, F. Fikes and E. Sandewall (eds), *Principles of Knowledge Representation and Reasoning: Proceedings of the Second International Conference,* Morgan Kaufmann, CA; pp. 417-428, 1991. A revised and exteneded version of this paper can be found in Peter Gärdenfors ed. *Belief Revision*, CUP, pp. 52-88, 1992.

[12] Rott, Hans "A Nonmonotonic Conditional Logic for Belief Revision I", in A. Fuhrmann and M. Mourreau, eds., *The Logic of Theory Change,* Lecture Notes in Artificial Intelligence 465, Springer Verlag, pp. 135-183, 1991.

[13] Tennant, Neil, "Changing the Theory of Theory Change: Towards a Computational Approach", *British Journal of Philosophy of Science*, vol. 45, pp. 865-897, 1994.